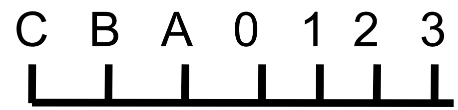
Negative numbers

In the lesson we learn how to use two manipulatives to check our work using alternative approaches. We find that many of the answers appear contrary to our intuitive expectations. We also should be learning how we build upon what we learn. We utilize our understanding of the number zero to apply it to solving adding and subtracting negative numbers. We learn how to make rules and then apply these rules to validate what we have been told. We should also be learning how to take notes, summarize what we learn and how to practice so that we can become fluent using negative numbers and retaing what we have learned.

We start by showing with pattern recognition, how we made have discovered negative numbers
We then show how to use negative numbers.
We then show how to prove what we do to help in the retention process.
Finally, we summarized what we learn and show expansion upon what we learn.

Discovering Negative Numbers



Creating our numbers

2=1+1

3=2+1

We create numbers by adding one to the previous number

1=1+nb1 number before 1 (usually zero but could be 12)

Using this we have

A+1=nb1=0 for now

A=0 – 1

A=-1

-1+1=0

B+1=A

B+1+1=A+1

B+2=0

B= 0 -2

B= -2 notation

-2 + 2 =0 substitution

Generalization and definition: -n+n=0

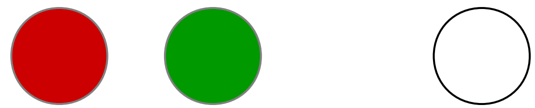
When we add we add a negative number to its additive inverse, we get the identity element for addition (and subtraction).

0+1 = 1	Identity property
0 = 1 – 1	subtraction
0+1-1=1-1	subtract 1
0+(1-1)=1-1	associative property
0 + 0 =0	transitive property
0=0 – 0	subtraction
0=-0	notation

Additional knowledge

0 and -0 occupy the same position on the number line

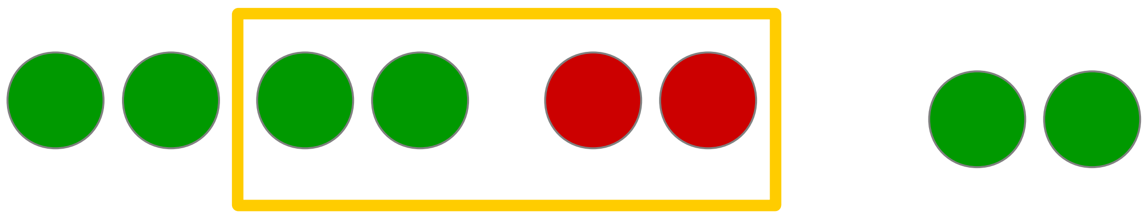
Negative Numbers



The RULE

-1 + 1 = 0

ADDITION

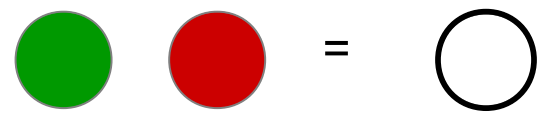


4 + - 2 = 2

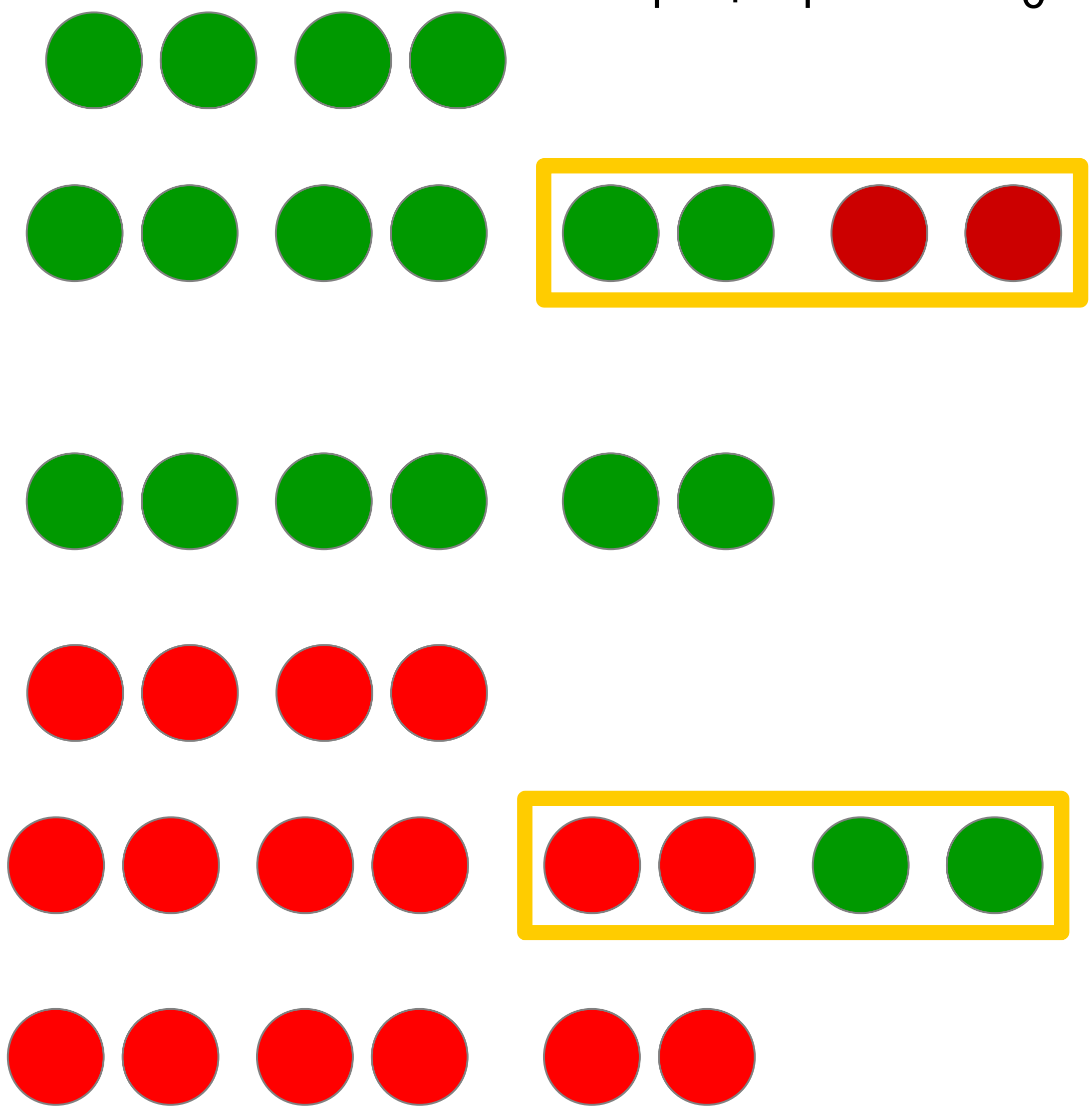
4 - 2 = 2

Adding a negative number is the same as subtracting a positive number + - = - + = -

Taking away what's not there!



1 + -1 = 0



Can we take away -2

Add 2 zeros (still 4)

Now take away -2

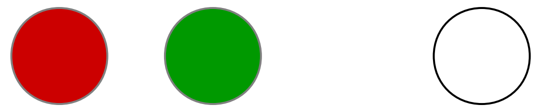
Can we take away 2

Add 2 zeros (still -4)

Now take away 2

The importance of the identity element

Negative Numbers

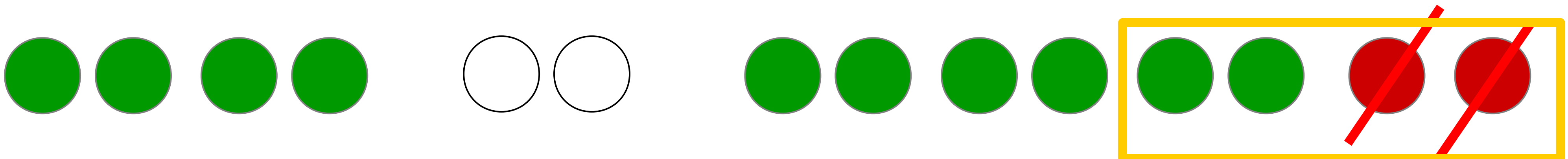


The RULE

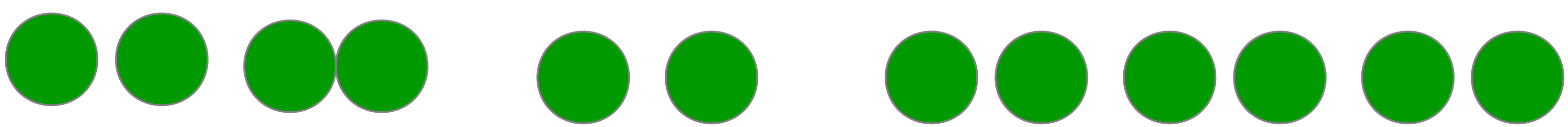
-1 + 1 = 0

Subtraction

Added 0



4 - - 2 = 6



Commutative Property

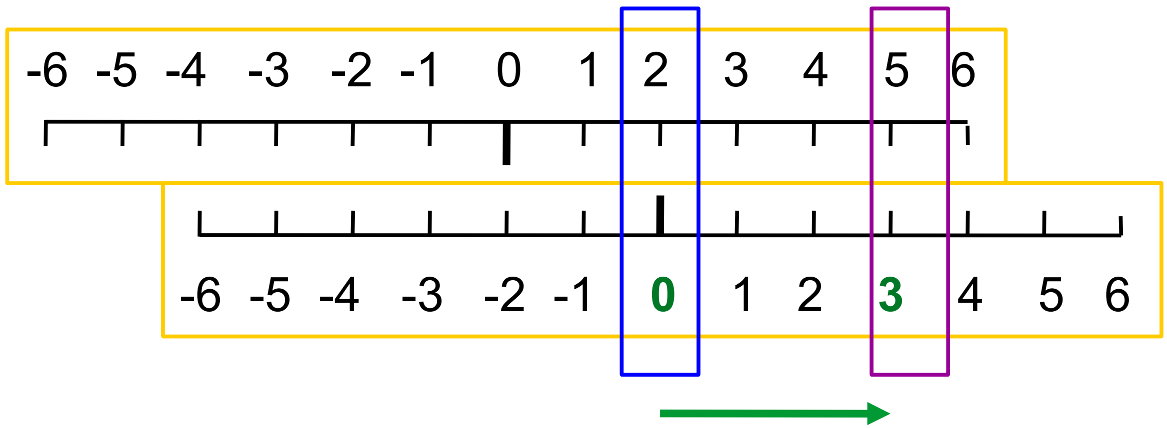
4 + 2 = 6

Subtracting a negative number is the same as adding a positive number (- - = +)

Using Double Rulers to Add and Subtract

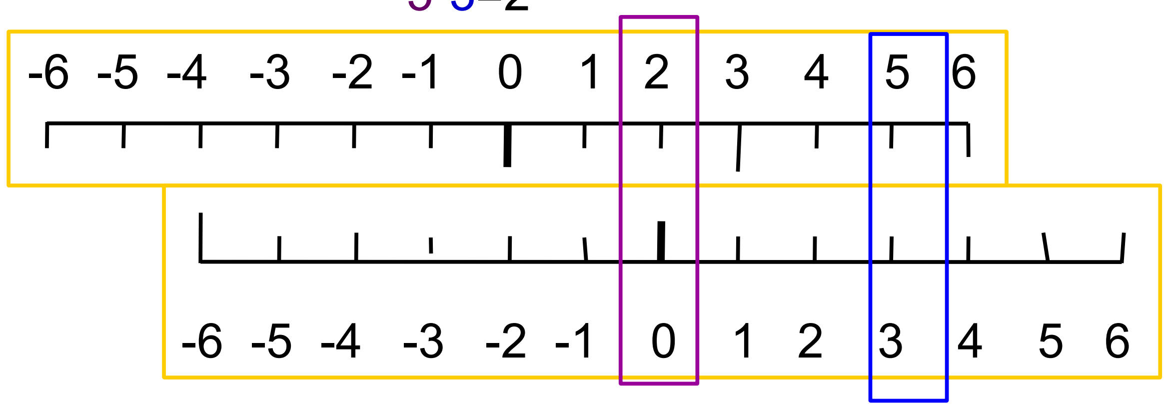
To add place the zero of the **bottom** ruler under the number on the **top** ruler.
On the bottom ruler look at number you wish to add; answer above it on top ruler.

2 + 3 = 5



To subtract place the number on the **bottom** ruler under the number on the **top** ruler.
On the bottom ruler look at zero; answer above it on top ruler.

5-3=2



4 – 2 =2

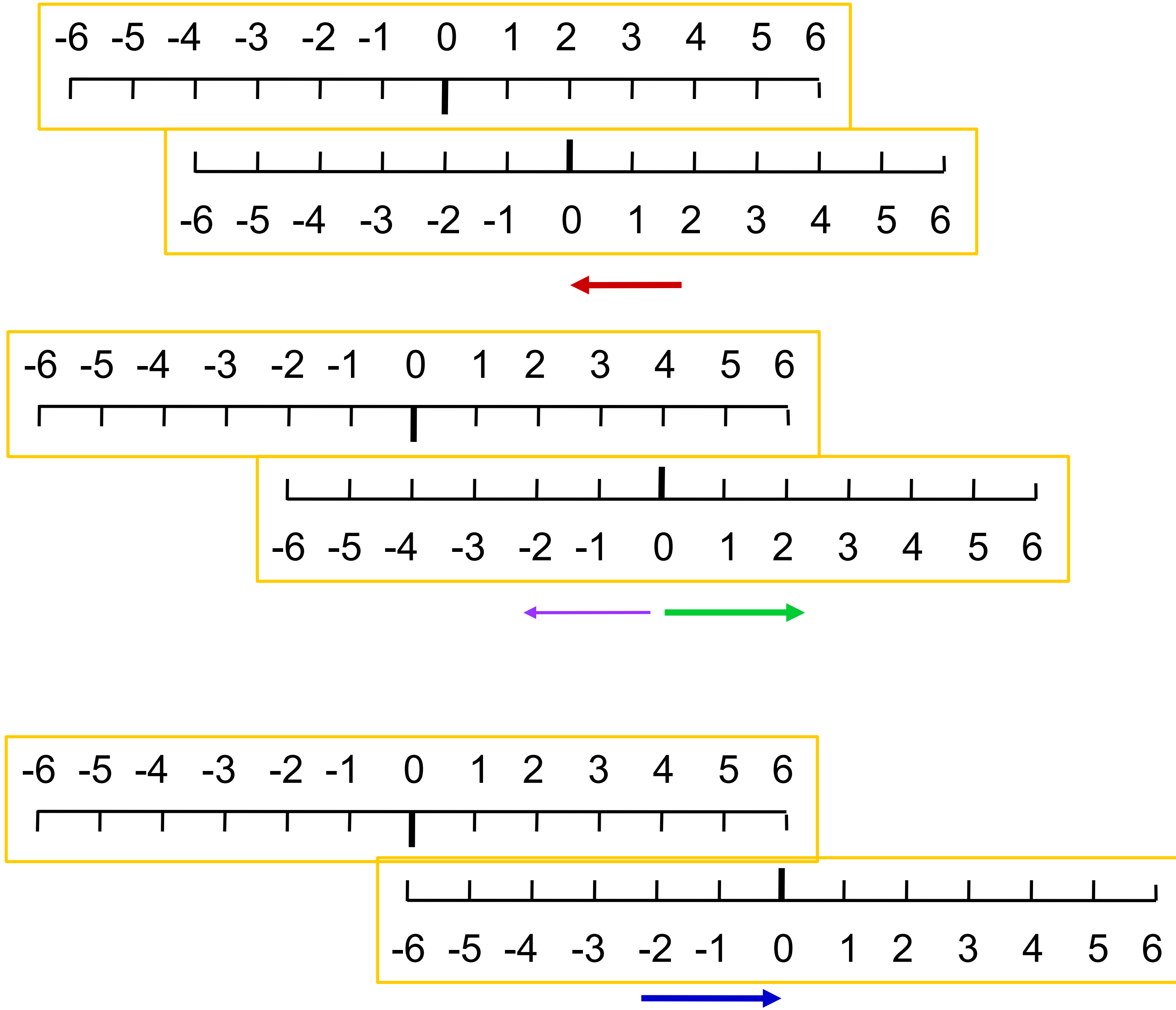
- = + -

4 + -2 =2

4 +2 = 6

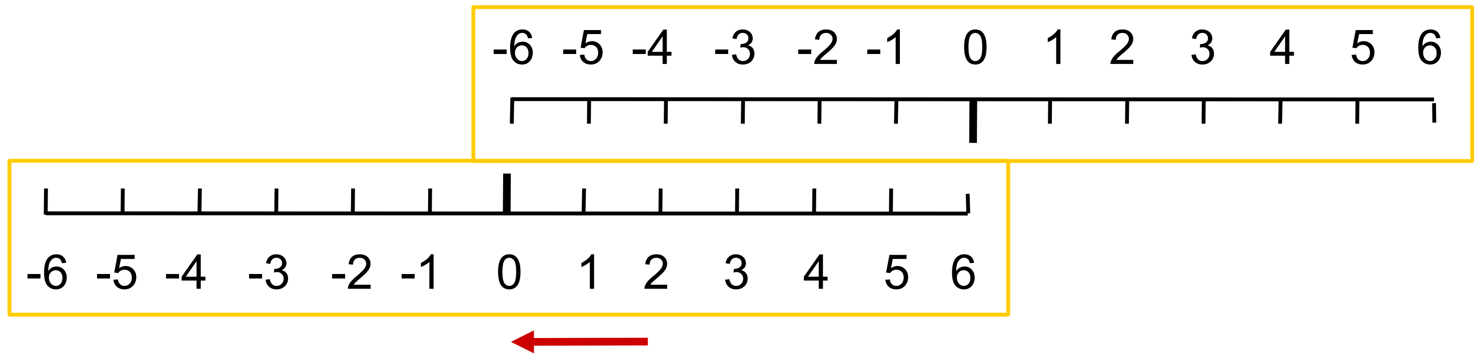
- - = =

4 - -2 = 6



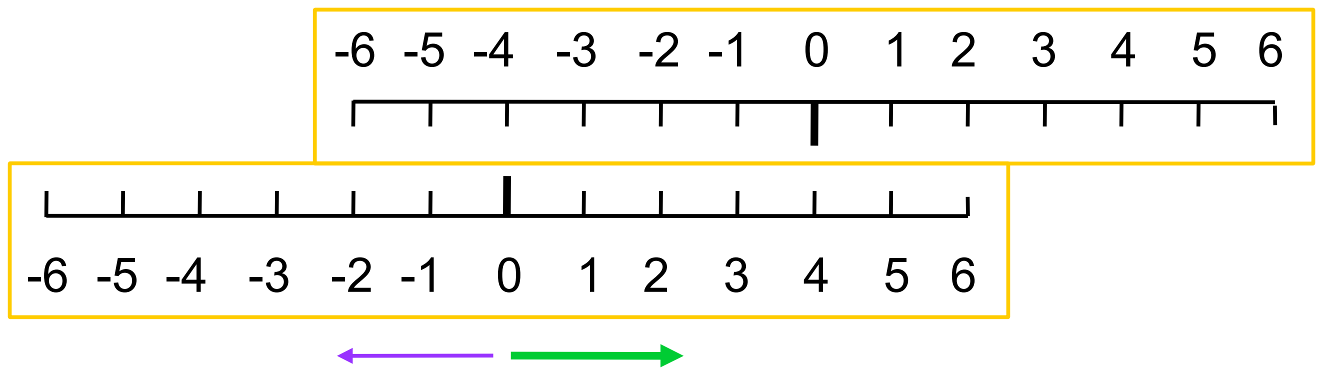
Completing examples

-4 – 2 =-6

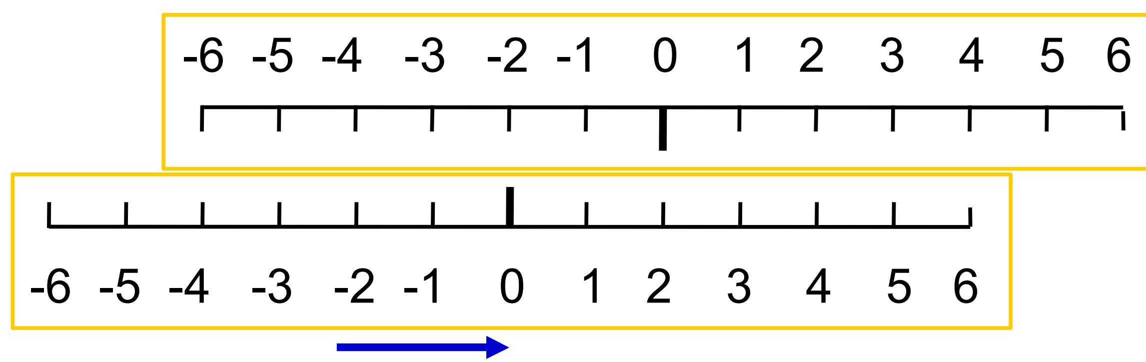


-4 + -2 =6

-4 +2 = -2



-4 - -2 = -2



$-1 + 1 = 0$ Definition of negative numbers

$\mathbf{-1 = 0 - 1}$ Subtraction (a)

$-3 + 3 + -2 + 2 = 0 + 0 = 0$ Definition

$3 + 2 + -3 + -2 = 0$ Commutative property

$(3 + 2) + (-3 + -2) = 0$ Associative property

$-3 + -2 = 0 - (3+2)$ Subtraction

$-3 + -2 = \mathbf{-(3+2)}$ Transitive property (a)

When we add negative numbers,
we add them as positive numbers and make sign negative.

Algebraic Approach

$-n + n = 0$ Definition of negative numbers

$\mathbf{-n = 0 - n}$ Subtraction (a)

$-n + n + -m + m = 0 + 0 = 0$ Definition

$n + m + -n + -m = 0$ Commutative property

$(n + m) + (-n + -m) = 0$ Associative property

$-n + -m = 0 - (n+m)$ Subtraction

$-n + -m = \mathbf{-(n+m)}$ Transitive property (a)

In this case where $n=3$ $m=2$

When we add negative numbers,
we add them as positive numbers and make sign negative.

The numeric and manipulative approach

$-1 + 1 = 0$

$\mathbf{-1 = 0 - 1}$

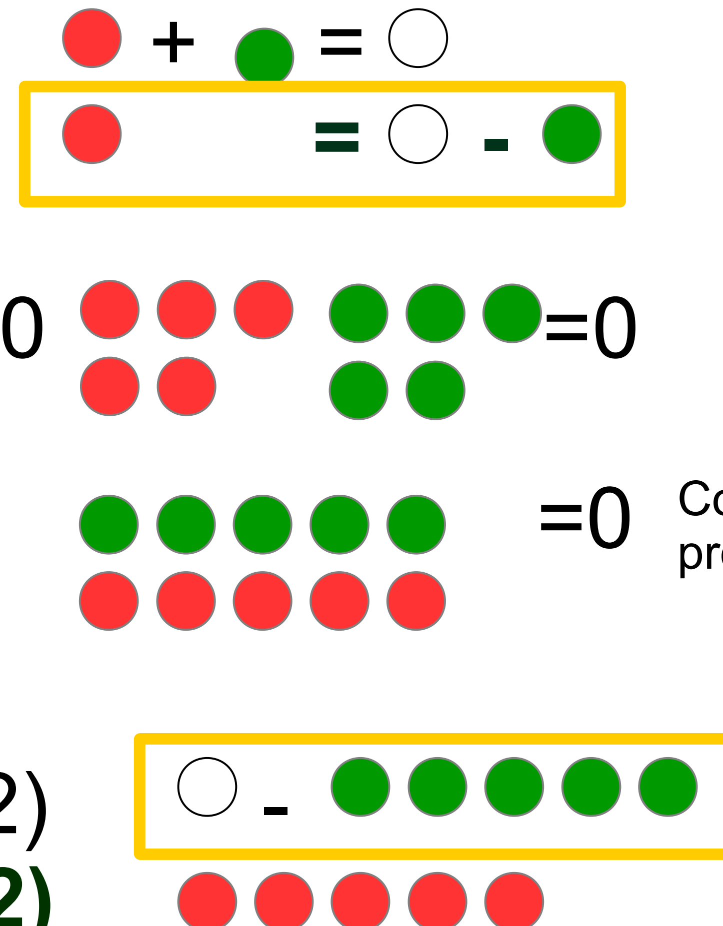
$-3 + 3 + -2 + 2 = 0 + 0 = 0$

$3 + 2 + -3 + -2 = 0$

$(3 + 2) + (-3 + -2) = 0$

$-3 + -2 = 0 - (3+2)$

$-3 + -2 = \mathbf{-(3+2)}$



Commutative property

Summary

2=+2

+ + = + **- - = -**

2+4=6 -2 + -4 =-6

- = - + =+ -

6+ -2 = 6-2=4

6- +2 = 6-2=4

When subtracting a bigger number from a smaller, subtract smaller from bigger and use the sign of the bigger number.

-4 +2 = -(4- 2)= -2

-2 + 4 = (4- 2)= 2

Exercise: 4 + 2 = -4 + 2=

4 - -2 = **-4 - -2=**

4 - 2 = **-4 - 2=**

4 + -2 = -4 + -2=

Three additional proofs

$n+-n=0$ Definition
$n=0 -n$ Subtraction (a)
$-n=0 - -n$ Subtraction (b)

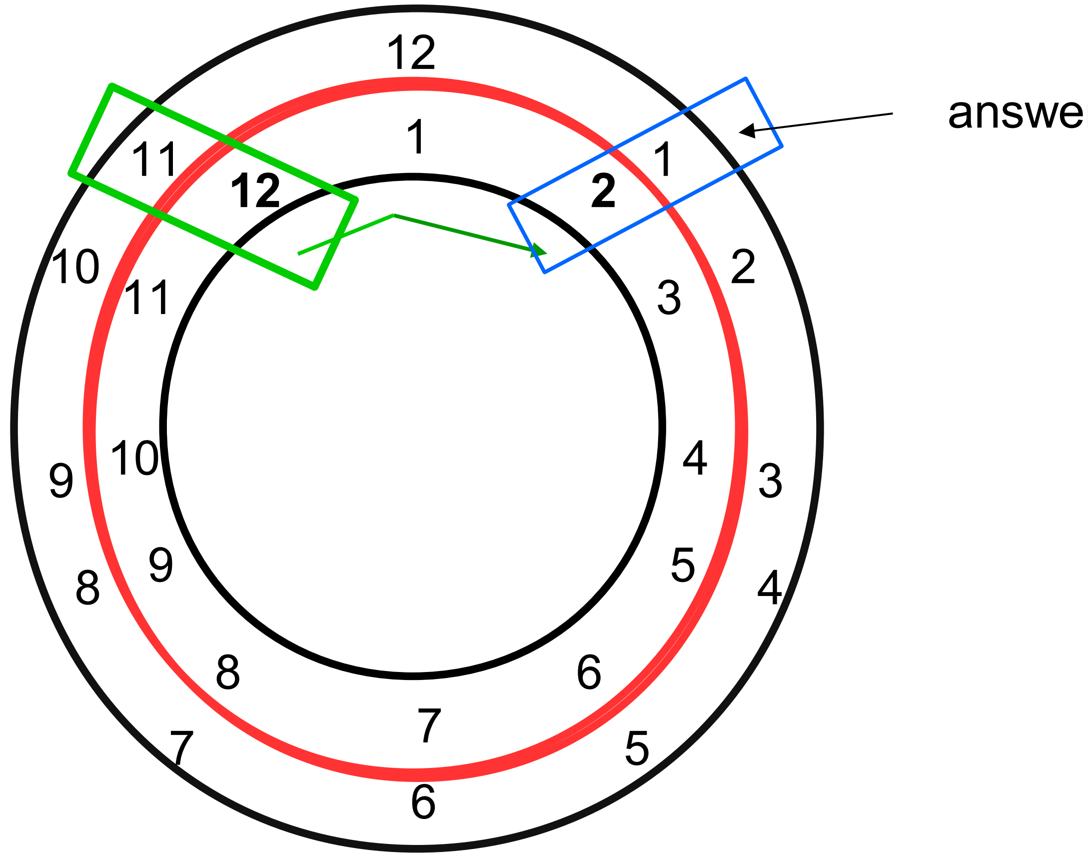
$n+0=n$ Identity property
$n+ 0-m =n-m$ subtraction
$n+(0-m) =n-m$ Associative property
$n+ -m =m-m$ transitive property (a)
Add a negative number is the same as subtracting a positive number

$n+0=n$ Identity property
$N+ 0 - - n =n - - m$ subtraction
$n+(0 – -m)=n - - m$ associative property
$N + m = n - - m$ transitive property (b)

Adding a positive number is the sam as subtrating a negative number
$-n + n + -m + m =0+0=0$ Definition
$m + -n +n + -m =0$ commutative property
$(m + -n)=0 – (n + -m)$ associative property
$m-n =0 - (n-m)$ Subtraction
$m-n = -(n-m)$ transitive property (a)
If $m=2$ $n=5$, then
$2-5=-(5-2)$
When subtracting a larger number from a smaller, shange order and make answer negative.

Modular (clock arithmetic)

Rotate inner circle counter clockwise. Place 12 under 11 and look at 2 in inner circle.



Twelve is the identity element: 12+5=5
Adding eleven is like subtracting 1

Circle	Line
12+1= 1	0+1=1
11+1=12	-1+1=0
11=12 -1	-1= 0 -1
11+2=1	-1+2=1

More advanced:
Line: -1 x-1=1
Circle 11x11=121=1